

Electron in the Ground Energy State—Part 4, Revision 1

Rotating Fields, Size of the Small Radius, and Electromagnetic Properties

David L. Bergman

Common Sense Science, PO Box 767306, Roswell GA 30076-7306

E-mail: bergmandavid@comcast.net

Dennis P. Allen, Jr.

17046 Lloyds Bayou Drive, #322, Spring Lake MI 49456-9273

E-mail: allens10@sbcglobal.net

Abstract for Part 4. Recent experiments that localize the seat of electric induction in a unipolar disk generator lead to the conclusion that the electric field of a spinning charged ring is rotating and thereby doubling the potential energies in the equations of the electron ring model in the ground-energy state. The exact size of the electron small radius r is obtained from the boundary conditions of a spinning charged ring (SCR). Fundamental and physical properties of the free, spinning charged ring electron in the ground-energy state are computed and shown in a revised table of Fundamental and Physical Properties of the SCR Model (May 2013) and a new table of System and Subsystems Properties of the SCR Model of a Free Electron.

Introduction. Recent research indicates that two refinements to the Spinning Charged Ring (SCR) model more accurately portray the free electron in the ground-energy state. Achilles declares that “Semi-classical electron models deserve to be thoroughly revisited in light of recent relevant experimental evidence on ‘rotating fields’” [34].

Previously, structural and electrical properties of a spinning ring electron were evaluated using an approximate value of the small radius r taken from reference [14]. But here, radius r is derived by applying continuity and equilibrium conditions on the charge density at the surface boundary of the SCR model of the electron.

ENERGY IN ROTATING FIELDS

Do Magnetic Fields Rotate With Their Source? In connection with his unipolar experiments, Michael Faraday raised the question of whether or not the magnetic field lines of a cylindrical magnet rotated with the magnet as the magnet is rotated about its line of symmetry; and he famously said “no,” because the sun rotates about its axis, but the rays of the sun do not rotate. But Steinmetz said that it didn’t matter as one gets the same answer whether or not they rotate. Wesley, however, states that “It is usually claimed that the magnetic B-lines do not rotate with the magnet, which gives the B-lines a mysterious nonphysical property.... Müller’s brilliant, but elementary method... [showed] that *the seat of the emf [electromotive force] remains in the disc [where the moving charge resides].*” [35, pp. 158-159]. Wesley then goes on to completely explain unipolar induction on the basis that the B-fields do rotate, under certain conditions, *e.g.*,

an electron in an excited-energy state. In addition, the experimentally observed result of Achilles and Guala-Valverde also indicates that the B-fields rotate so that the result is a 'relative angular velocity result' [34, equation (1)].

However, while rotation of the electric field applies to the ground-energy state, it seems that *rotation of the magnetic field applies only to the excited-energy states*. Let the ring have its point of symmetry at the origin of a rectangular Cartesian coordinate system and assume its plane of symmetry is the x-y plane. The charge is rotating at a linear velocity of c . Next, rotate the x-y plane about the z-axis at an angular velocity ω . Now, the charge will rotate at a new angular velocity of $\omega' = c/R + \omega$. But, the Achilles's result says that the magnetic field is rotating about the z-axis at angular velocity ω , whence the electron without the x-y plane rotating about the origin must not have a rotating magnetic field.

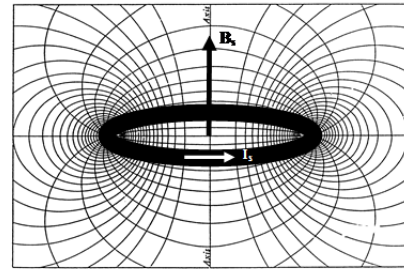


Figure 10.

Non-Rotating Magnetic B-Field encircles charge moving at the speed of light (current I_s).

Energy of a Non-Rotating Magnetic B-Field. For a non-rotating magnetic field (see Figure 10), spin current of electron charge circulating at the speed of light produces a magnetic pinch force that compresses the ring to an outside classical magnetic potential energy:

$$\mathcal{E}_{so} = - \frac{c^2 \mu_0 q^2 \left(\log \frac{8R}{r} - 2 \right)}{8\pi^2 R} \quad (86)$$

Additional Energy from Rotating Electric Field. As shown by Francisco Müller [36], the seat of electromotive force (emf) is in the charge located at the surface of the copper disk in Figure 11—and located inside a steady-state, ground-energy electron that is modeled as the SCR model. The Appendix provides a rigorous derivation showing that including the unipolar induction doubles the total energy in the spinning charged ring. Thus, the electromagnetic energies are all doubled in the spinning charged ring.

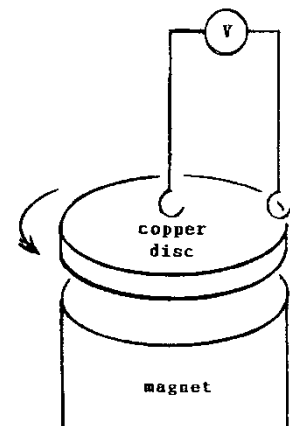


Figure 11. The essential elements involved in unipolar induction.

Applying the Achilles and Guala-Valverde Result to the SCR Model. In previous electron models the magnetic potential energy of a spinning charge ring has been taken without including the induction energy resulting from rotation of the electric field. This error of omission is corrected here by entering a factor of two (2) that doubles the potential energy in the following equations:

Outside magnetic potential	\mathcal{E}_{so}	Equation (31 in Part 1 and equation (86) in Part 3).
Outside electric potential	\mathcal{E}_{eo}	Equation (35 in Part 1 and equation (87) in Part 3).
Inside magnetic potential	\mathcal{E}_{si}	Equation (88 in Part 3).
Inside electrical potential	\mathcal{E}_{ei}	Equation (88 in Part 3).

Thus, these four equations must be modified and become:

$$\mathcal{E}_{so} = -\frac{c^2 \mu_o q^2 \left(\log \frac{8R}{r} - 2 \right)}{4\pi^2 R} \quad (95)$$

$$\mathcal{E}_{eo} = \frac{c^2 \mu_o q^2 \left(\log \frac{8R}{r} \right)}{4\pi^2 R} \quad (96)$$

$$\mathcal{E}_{si} = \frac{c \left(-2\hbar\pi^2 - 2c\mu_o q^2 + c\mu_o \log \left[\frac{8R}{r} \right] \right)}{4\pi^2 R} \quad (97)$$

$$\mathcal{E}_{ei} = -\frac{c \left(-2\hbar\pi^2 - 2c\mu_o q^2 + c\mu_o \log \left[\frac{8R}{r} \right] \right)}{4\pi^2 R} \quad (98)$$

SIZE OF THE ELECTRON SMALL RADIUS

Previously, in Part 3, the structural and electrical properties of a spinning ring electron were modeled by using an approximate value of the small radius r taken from reference [14]. But here, radius r is derived by applying three boundary conditions on the charge density $\lambda[\rho]$, defined at equation (18), at the surface of the SCR model of the electron:

$$\lambda[\rho] \equiv \frac{r b_{res}}{\rho} + b_o + \frac{b_1 \rho}{r} + \frac{b_2 \rho^2}{r^2} \quad (18)$$

Specifically, (1) the electron charge density must be finite everywhere *inside* the ring, (2) the electron charge density must be zero everywhere *outside* the ring, and (3) to ensure a smooth, continuous function and avoid a discontinuity at the *surface boundary* of the ring electron, the second derivative of electron charge density must be set to zero at the rim where $\rho = r$.

$$\lambda'' \equiv \frac{\partial^2 \lambda[\rho]}{\partial \rho^2} = \frac{2b_2}{r^2} + \frac{2rb_{res}}{\rho^3} \quad (99)$$

(Previously, in Part 2, to help ensure smoothness in $\lambda[\rho]$ at the rim where $\rho = r$, we set $\lambda'_{\rho=r} = 0$ when solving for the b_2 -coefficients by using the first derivative of electron

charge density. The distribution of charge density inside the ring is still constrained here by the boundary condition imposed earlier that $\lambda'_{\rho=r} = 0$).

At the surface of the charged ring where $\rho = r$,

$$\lambda'' = \frac{2b_2}{r^2} + \frac{2b_{res}}{r^2} \quad (100)$$

where b_2 was replaced by b_{212} and then evaluated using the fundamental physical constants listed in Table 1, and where b_{res} was replaced by b_{res12} and then evaluated using the fundamental physical constants listed in Table 1. Then, by setting λ'' equal to zero, a second order equation in terms of the small radius r was created and then solved. Thus, for an electron with electric field rotating about the axis of symmetry, and a smooth boundary, two values of the small radius r were computed and two mathematical solutions were found: $r_1 = 3.46425235(134) \times 10^{-106}$ meter and $r_2 = 5.86342783(830) \times 10^{-106}$ meter.

Next, for both of the two new values of the small radius, the fundamental and physical properties of the spinning charged ring electron in the ground-energy state were re-computed. The re-computed properties remain the same for either value of the small radii r (including Property #9, Total Potential Energy, \mathcal{E}) and are shown in Table 3, Fundamental and Physical Properties of the SCR Model (May 2013).

PHYSICAL AND ELECTRICAL PROPERTIES

Magnetic Flux Outside the Ring Electron. One unit of charge e moving around the circumference of the electron ring with velocity c creates a loop current I_s [14, equation 4]:

$$I_s = \frac{ec}{2\pi R} \quad (101)$$

The magnetic flux outside the ring for this current is

$$\phi_{o1} = \frac{2\mathcal{U}_{so1}}{I_s} = -4.1179421271071106595 \times 10^{-15} \text{ Wb} \quad (102)$$

where \mathcal{U}_{so1} is the outside magnetic rest-energy for the case that small radius $r = r_1$.

Likewise,

$$\phi_{o2} = \frac{2\mathcal{U}_{so2}}{I_s} = -4.1078316213085771068 \times 10^{-15} \text{ Wb} \quad (103)$$

where \mathcal{U}_{so2} is the outside magnetic rest-energy for the case that small radius $r = r_2$.

Magnetic Flux Inside the Ring Electron. One unit of charge e moving around the circumference of the electron ring with velocity c creates a loop current I_s [14, equation 4]:

$$I_s = \frac{ec}{2\pi R} \quad (101)$$

The magnetic flux inside the ring for this current is

$$\phi_{i1} = \frac{2\mathcal{U}_{si1}}{I_s} = -1.77252065883993108 \times 10^{-17} \text{ Wb} \quad (104)$$

where \mathcal{U}_{si1} is the inside magnetic rest-energy for the case that the small radius $r = r_1$.

Likewise,

$$\phi_{i2} = \frac{2\mathcal{U}_{si2}}{I_s} = -2.78357123825318635 \times 10^{-17} \text{ Wb} \quad (105)$$

where \mathcal{U}_{si2} is the inside magnetic rest-energy for the case that small radius $r = r_2$.

Total Magnetic Flux for the Case that $r = r_1$. Adding equations (102) and (104) gives the total magnetic charge for the case that the small radius $r = r_1$.

$$\phi_1 = \phi_{o1} + \phi_{i1} = -4.135667333691108970 \times 10^{-15} \text{ Wb} \quad (106)$$

Total Magnetic Flux for the Case that $r = r_2$. Adding equations (103) and (105) gives the total magnetic charge for the case that the small radius $r = r_2$.

$$\phi_2 = \phi_{o2} + \phi_{i2} = -4.135667333691108970 \times 10^{-15} \text{ Wb} \quad (107)$$

From equations (106) and (107), the magnetic charge $\phi = \phi_1 = \phi_2$ remains constant in the SCR model and in nature—just as the electric charge e also remains constant in the SCR model and in nature.

Planck's Constant. Planck's constant (denoted as h) is a physical constant that holds true for any rotating system of charged particles. In the case of a spinning charged ring electron, Planck's constant is

$$h = e\phi = 6.6260689633 \times 10^{-34} \text{ J} \cdot \text{s} \quad (108)$$

where e is the electric charge, and ϕ is the magnetic charge just computed. The SCR model of the electron accurately reflects the value of Planck's constant observed in various physical phenomena. (Some other equations use the 'reduced Planck's constant' $\hbar \equiv h/2\pi$.)

Fundamental Properties of the SCR Model of a Free Electron. Table 3 shows that the SCR model yields realistic properties of a 'free electron' which are consistent with causality, logic, conservation of energy, laws of electromagnetics, and empirical data.

Property # Solution #	Property	Value	Data/Equation
1	Electric charge, e	$-1.602176487(40) \times 10^{-19}$ Coulomb	CODATA-2006, Table 1
2	Mass, $\mathcal{M} = \mathcal{U}/c^2$	$9.10938215(45) \times 10^{-31}$ kg	CODATA-2006, Part 3 Equation (90)
3	Large radius, R	$3.87953220(795) \times 10^{-13}$ meter	Part 2 Equation (66)
4-1	Small radius, r_1	$3.46425235(134) \times 10^{-106}$ meter	Part 3 Equation (85)
4-2	Small radius, r_2	$5.86342783(830) \times 10^{-106}$ meter	
5	Velocity of loop charge, v_s	c (speed of light) meter/second	CODATA-2006, Part 1 Equation (29)
6	Magnetic moment, μ_e	$-9.29555099(050) \times 10^{-24}$ J/T	Part 2 Equation (67)
7	Angular momentum, p_s	$-5.27285814(265) \times 10^{-35}$ J s	CODATA-2006, Part 2 Equation (57)
8	Gyromagnetic ratio, γ_e	$1.76290557(019) \times 10^{11} \text{ s}^{-1} \text{ T}^{-1}$	Part 3 Equation (94)
9	Total potential energy, \mathcal{E}	$3.78584540(069) \times 10^{-16}$ J	Part 3 Equation (86)
10	Total magnetic flux, ϕ	$-4.13566733(369) \times 10^{-15}$ Wb	Part 4 Equations (106) and (107)
11	Planck's constant, $h = e \phi$	$6.62606896(175) \times 10^{-34}$ J s	Part 4 Equation (108)

Table 3.
Fundamental and Physical Properties of the SCR Model of the Electron (May 2013)

Notes on the Properties Shown in Table 3. The SCR model has been adapted to conform to the empirical data of electron charge, electron mass, and the fundamental physical constants reported in CODATA-2006 and listed in Table 1. This model with rotating electric fields produces the actual properties listed in Table 3 of a *free* electron in the *ground-energy state*.

The computations in Table 3 are shown with twelve digits precision although the working precision for computations was usually much greater than twelve digits. However, the accuracy of the properties shown in Table 3 might not exceed nine digits, i.e., the minimum accuracy of the fundamental constants of Table 1.

Specific details about each of the eleven physical and electrical properties are listed below:

1. **Electric charge, e .** Physicists and electrical engineers consider the electric charge carried by the electron to be a fundamental physical constant. The SCR model of the electron makes the reasonable assumption that this charge is the actual material substance contained within the boundaries of ring electron particles. Every electron in nature is composed of one unit of charge equal to the value shown for property #1 of Table 3.

2. **Mass, \mathcal{M} .** The SCR model of the electron produces the empirical value of mass reported in CODATA-2006.
3. **Large radius, R .** The SCR model of the electron produces a real, finite-size ‘physical object’ with radius R shown in Table 3.

The Standard Model insists that the electron is a *point-particle* and asserts that the electron is a “quantum object” with inherent properties of mass, charge, magnetic moment, and angular moment (spin). But compression of an electron’s charge to a point would require an infinite amount of force and mass-energy—a scientific fact that was intentionally ignored by Dirac [29] and that is still ignored by contemporary particle physicists. The point-particle assertion is made in spite of empirical evidence for finite-sized particles, and in violation of the well-established, fundamental laws of classical physics, especially the ‘law of conservation of mass and energy.’

Arthur H. Compton wrote in 1919 that “the experimental observations on the scattering of high frequency radiation by matter could be explained only on the hypothesis that the radius of the electron is comparable with the wave-length of hard γ -rays. The phenomena of scattering were found to be quantitatively accounted for, within the probable errors of observation, if the electron was considered to be a flexible ring of electricity with a radius of 2×10^{-10} cm” [30].

4. **Small radius, r .** The SCR model of the electron produces a real, finite-size ‘physical object’ with two mathematical solutions for radii r shown in Table 3 (May 2013). This computation, equation (85), of *two* possible values of the small radius r , is derived from the SCR model with interior charge and energy included. Either one or both dimensions might be the actual size of the electron small radius r . For the same reasons given above for rejecting the point-particle conjecture of the large radius R (two paragraphs above), neither can the real dimension of the small radius r be construed to be actually zero.¹
5. **Velocity of loop charge, v_s .** As shown in Part 1, distributed charge located inside the spinning ring generates electrostatic pressure that is in equilibrium with magnetostatic pressure throughout the interior *only when the charge velocity along the ring circumference is at the speed of light*.
6. **Magnetic moment, μ_e .** The magnetic moment μ_e of a loop of current equals the product of its area A by its current I so that $\mu_e = AI = \pi R^2 I$ [19, p. 280].

¹ Although it certainly is true that the electron and proton cannot be point-particles, it is nevertheless also true that the point-particle *approximation* has led to nuclear weapons (for example) which work very well indeed (too well, in fact!), and so the point-particle model cannot be too far off either (especially under certain conditions), whence an extremely small value of the small radius r cannot be unexpected!

7. **Angular momentum, p_s .** The SCR model of the electron uses the empirical value reported in CODATA-2006. Here, the ring model has been conformed to the undisputed experimental data.
8. **Gyromagnetic ratio, γ_e .** The “best” measurements of the gyromagnetic ratio are made on single electrons captured and held by a Penning trap composed of electric and magnetic fields [31]. These fields couple with the *trapped* electron to affect and change its size and the measured value of gyromagnetic ratio.

No high-precision gyromagnetic ratio measurement of a *free* electron (in contrast to a *trapped* electron) has been performed, but an expected value is predicted here. Of course, the value predicted at equation (94) and the CODATA-2006 value of a trapped electron are different because the ambient environment is different (i.e., *free* versus *containment* in a Penning trap).

9. **Total potential energy, \mathcal{E} .** Total potential energy \mathcal{E} carried by an electron is the algebraic sum of four components of field energies, electric and magnetic, located both inside and outside an electron. An electron self-adjusts its physical dimensions to the size that locally minimizes its total potential energy.
10. **Total magnetic flux, ϕ .** The flux generated by a spinning charged ring is the actual magnetic flux and magnetic charge of every free electron, i.e., the “magnetic quantum.” It is twice the magnetic charge given by CODATA-2006 which erroneously defines the magnetic quantum as $\phi \equiv h/2e$ instead of $\phi \equiv h/e$.
11. **Planck’s constant, $h = e \phi$.** The magnitude of Planck’s constant is known to have a value equal to the product of the electric charge q times its magnetic charge Φ_0 for any rotating system of charge. Thus the SCR model, a rotating system of charge, conforms with Planck’s constant (as it must in order to be credible) since its electric charge q equals the electric quantum e and its magnetic charge ϕ equals the magnetic quantum Φ_0 .

System and Subsystems Properties. Complete system and subsystems properties of the free electron are given in Table 4 for both values of the small radius r .

Electron Property	Property value if	Property value if
	small radius $r = r_1$ $r_1 = 3.46425235(134) \times 10^{-106}$	small radius $r = r_2$ $r_2 = 5.86342783(830) \times 10^{-106}$
Electric charge, e	$-1.602176487(40) \times 10^{-19}$ Coulomb	$-1.602176487(40) \times 10^{-19}$ Coulomb
Magnetic charge, ϕ	$-4.135667333(69) \times 10^{-15}$ Weber	$-4.135667333(69) \times 10^{-15}$ Weber
Inside Magnetic Charge, ϕ_i	$-1.77252065(840) \times 10^{-17}$ Weber	$-2.78357123825 \times 10^{-17}$ Weber
Outside Magnetic Charge, ϕ_o	$-4.11794212(711) \times 10^{-15}$ Weber	$-4.10783162(131) \times 10^{-15}$ Weber
Planck's Constant, $h = (e \phi)$	$+6.62606896(175) \times 10^{-34}$ J s	$+6.62606896(175) \times 10^{-34}$ J s
Outside electric potential energy, \mathcal{E}_{eo}	$+4.09501784(591) \times 10^{-14}$ J	$+4.08505657(587) \times 10^{-14}$ J
Outside magnetic potential energy, \mathcal{E}_{so}	$-4.05715939(191) \times 10^{-14}$ J	$-4.04719812(186) \times 10^{-14}$ J
Inside electric potential energy, \mathcal{E}_{ei}	$+1.74635743(160) \times 10^{-16}$ J	$+2.74248443(609) \times 10^{-14}$ J
Inside magnetic potential energy, \mathcal{E}_{si}	$-1.74635743(160) \times 10^{-16}$ J	$-2.74248443(609) \times 10^{-14}$ J
Inside potential energy, \mathcal{E}_i	0	0
Total potential energy, \mathcal{E}	$+3.78584540(069) \times 10^{-16}$ J	$+3.78584540(069) \times 10^{-16}$ J
Outside electric rest-mass energy, \mathbf{u}_{eo}	$+4.09501784(591) \times 10^{-14}$ J	$+4.08505657(587) \times 10^{-14}$ J
Outside magnetic rest-mass energy, \mathbf{u}_{so}	$+4.05715939(191) \times 10^{-14}$ J	$+4.04719812(186) \times 10^{-14}$ J
Inside electric rest-mass energy, \mathbf{u}_{ei}	$+1.74635743(160) \times 10^{-16}$ J	$+2.74248443(609) \times 10^{-16}$ J
Inside magnetic rest-mass energy, \mathbf{u}_{si}	$+1.74635743(160) \times 10^{-16}$ J	$+2.74248443(609) \times 10^{-16}$ J
Inside rest-mass energy, \mathbf{u}_i	$+3.49271486(320) \times 10^{-16}$ J	$+5.48496887(218) \times 10^{-16}$ J
Total rest-mass energy, \mathbf{u}	$+8.18710438(645) \times 10^{-14}$ J	$+8.18710438(645) \times 10^{-14}$ J
Values of the b -coefficients in the Laurent Equation normalized to a positive unit volume charge density inside the ring structure.	Normalized $b_{res12} = -0.7096774193$ Normalized $b_{012} = -1.74193548710$ Normalized $b_{112} = +5.61290322581$ Normalized $b_{212} = -3.1612903226$	Normalized $b_{res12} = -2$ Normalized $b_{012} = +6$ Normalized $b_{112} = -6$ Normalized $b_{212} = +2$

Table 4. System and Subsystems Properties of the SCR Model of a Free Electron.

Notes on the Properties Shown in Table 4. Every electron is a miniature *system* of a material substance, electric charge, configured as a spinning charged ring. Table 3 describes and quantifies the system properties of the electron, and Table 4 describes and quantifies electrical and magnetic subsystems that function inside and outside electrons.

Table 4 gives the actual electromagnetic properties of a ring electron and its subsystems showing the partitioning of electromagnetic energy both inside and outside the ring electron.

The SCR model has been adapted to conform to the empirical data of electron charge, electron mass, Planck's constant, and the fundamental physical constants reported in CODATA-2006 and listed in Table 1. This model with rotating electric fields provides the actual properties listed in Table 4 of a *free* electron in the *ground-energy state*.

The computations in Table 4 are shown with twelve digits precision although the working precision for computations was usually much greater than twelve digits. However, the accuracy of the properties shown in Table 4 might not exceed nine digits, i.e., the minimum accuracy of the fundamental constants of Table 1.

Other Electron Properties. In addition to the fundamental and physical properties of a free electron in the ground energy state, electron properties have been measured in various environments with corresponding data collected, e.g., Compton wavelength, photoelectric effect, line spectra, and the important double-slit interference experiments. Some previous studies by Common Sense Science [37] appear to show that the flexible ring model known as the spinning charged ring is able to account for all of the experimental observations of electron properties.

Acknowledgements. We acknowledge the help of the Holy Spirit of God, Who is the Spirit of Truth and the source of all Wisdom and Knowledge, without Whom this work on the electron ring model could not have been successfully accomplished [21]. Charles W. Lucas, Jr., explained that deformation of elastic objects is the mechanism for exchange of energy between charged particles. We thank Thomas G. Barnes (deceased) for his pioneering work on *physical models of the electron* [22], and for showing that accelerating a charged particle modifies the electric and magnetic fields that surround the particle and *produce a self-force such as the force of inertia on the particle* [23]. We thank Glen C. Collins who proof-read and edited this paper. Finally, the second author would refer the reader to [21] in connection with the above.

References and Notes for Part 4. For references [1] through [23] see Part 1. For references [24] through [28] see Part 2. For references [29] through [33] see Part 3.

[34] R.A. Achilles, “Back to a Tenable Electron Model,” **Galilean Electrodynamics**, Volume 21, Number 6 (2010).

[35] J.P. Wesley, **Scientific Physics**, Benjamin Wesley, publisher (2002).

[36] J.P. Wesley (editor), **Progress in Space-Time Physics**, F.J. Müller (author) “Seat of Unipolar Induction” pp. 156-169, Benjamin Wesley, publisher (1987).

[37] Common Sense Science, www.CommonSenseScience.org.

Appendix, Revision 1

Double Electromagnetic Energy of a Spinning Charged Ring

Wesley's *Scientific Physics*¹ gives the formula for an electric potential energy E_s due to the scalar potential Φ , the last term of (4.119):

$$E_s = \frac{(\nabla\Phi)^2 + ((\partial\mathbf{A}/\partial t)/c)^2}{8\pi} \quad (\text{A1})$$

where Φ is the electrostatic scalar potential [reference 35, p. 125].

A dual formula holds (for M_s , the additional magnetic potential energy due to the new magnetic scalar potential Φ') for the magnetic fields including monopoles. Thus

$$M_s = \frac{(\nabla\Phi')^2 + ((\partial\mathbf{A}'/\partial t)/c)^2}{8\pi} \quad (\text{A2})$$

where Φ' is the magnetic scalar potential.

Wesley equation (4.117b) gives the traditional energy equation of the Maxwell theory E_p :

$$E_p = \frac{E^2 + M^2}{8\pi} \quad (\text{A3})$$

The last of equations (4.117) with $\mathbf{E} = -\nabla\Phi - ((\partial\mathbf{A}/\partial t)/c)$ when substituted for \mathbf{E} , and also (by duality) $\mathbf{M} = -\nabla\Phi' - ((\partial\mathbf{A}'/\partial t)/c)$, where \mathbf{A}' is the electric vector dual of the magnetic vector potential \mathbf{A} (just as Φ' is the magnetic dual of the electric potential Φ) then yields:

$$E_p = \frac{E^2 + M^2}{8\pi} = \left\{ \frac{(\nabla\Phi)^2 + ((\partial\mathbf{A}/\partial t)/c)^2 + 2(\nabla\Phi) \cdot ((\partial\mathbf{A}/\partial t)/c) + (\nabla\Phi')^2 + ((\partial\mathbf{A}'/\partial t)/c)^2 + 2(\nabla\Phi') \cdot ((\partial\mathbf{A}'/\partial t)/c)}{8\pi} \right\} \quad (\text{A4})$$

Thus, the total electromagnetic energy E is just then:

$$E = E_p + E_s + M_s \quad (\text{A5})$$

Since, for our steady (in time) state ring², the time partial derivatives all vanish identically, there remains only $(2E^2 + 2M^2)/8\pi$, that is, **double electromagnetic potential energy**.

¹J.P. Wesley, **Scientific Physics** (2002). In this appendix, the notations are left as Wesley selected for his book. The appendix uses the Scientific System of Units instead of the SI Units that are common today.

² Ampere's circuital law and the Lorentz law of force are wrong (in general) according to Wesley, but do hold for our steady state ring.