Electron in the Ground Energy State — Part 2

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Abstract for Part 2. Equilibrium of charge distributed throughout the electron ring interior is a result of electromagnetic self-forces that control the structure and internal motion of charge. The exact distribution of charge density inside the ring-shaped electron is described by equations and graphs. The dimensions of ground-state electrons are obtained from a physical spinning charged ring (SCR) model. The model yields the large radius and small radius of the electron ring. And the model also yields the actual non-anomalous magnetic moment of the electron. Thus, the SCR Model predicts that the diameter of a free electron is finite and physically related to electron properties such as its rest-mass, spin, magnetic moment, line spectra, and wavelength.

MASS[1] AND SELF-ENERGY OF AN ELECTRON

Consider a 'thought experiment' where work is done to assemble a charged ring composed of the substance called 'electrical charge.' Small pieces of this charge, called 'segments,' repel every other segment with a pressure given by Coulomb's Law. The work done $\mathcal{W}$ to assemble the charged ring resides inside the ring and its surrounding field as 'rest-mass energy' or 'mass-equivalent energy' or 'self-energy' $\mathcal{U}$. (Assembly of the charged ring is done very slowly in order to prevent any significant magnetic induction and any energy by radiation from entering into the computation of total ring energy.)

Since this ring consists of charge all of like sign (i.e., all negative charge for an electron), strong Coulomb forces should push it apart. What force could hold together this ring of compressed charge? Gravity is too weak. The 'strong nuclear force' acts only over ranges much less than the ring diameter, and the 'weak force' is said to cause neutron beta decay—not stability! Only *magnetic* force/pressure remains as a potential force to hold an electron together.

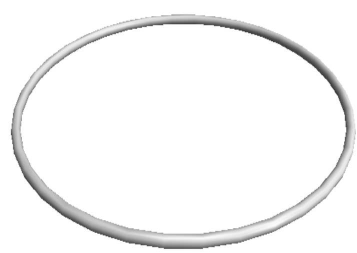


Figure 1. Electron Torus or Ring Structure

[1] The SCR model is a pure electrodynamics field model. The origin of mass is a result of electromagnetic field energy.

If the internal charge inside the ring is moving around the loop with a velocity *c* equal to the speed of light, then a magnetic “pinch pressure” would result from the charge in motion (i.e., a current). Part 1 shows that this force opposes the Coulomb pressure with equal intensity, i.e., at the surface and throughout the interior of the ring, the pressures from the electrostatic field and the magnetostatic field are balanced in magnitude and in opposite direction. Our computations of the two energy fields show that the spinning charged ring electron has slightly greater electrostatic field energy than magnetostatic field energy. While the energies are slightly different, the forces associated with each energy are equal and opposite in direction yielding a balanced particle structure in equilibrium.

Self-energy of the electron. The inertial mass of an electron $\mathcal{M}_{\text{elec}}$ is taken to be the amount of mass given by the so-called ‘Einstein relationship’:

$$\mathcal{U}_{\text{elec}} = \mathcal{M}_{\text{elec}}\, c^2 \qquad (45)$$

It is important to note that the self-energy embodied in the electron $\mathcal{U}_{\text{elec}}$ is not merely a collection of some potential energies (with positive or negative sign depending upon the force direction) but are also *real and always positive energies*. Like the self-energy of an electron, the equivalent energy of its inertial mass is always a *positive* quantity.

The self-energy $\mathcal{U}$ of an assembled system of components, including the electron, consists of all the mutual potential energies $\mathcal{E}$ (taken as positive energies) between the components. Unlike potential energies, self-energies are always positive, like mass. Thus,[2]

$$\mathcal{U}_{\text{ei}} = |\mathcal{E}_{\text{ei}}| \qquad (46)$$

$$\mathcal{U}_{\text{eo}} = |\mathcal{E}_{\text{eo}}| \qquad (47)$$

$$\mathcal{U}_{\text{si}} = |\mathcal{E}_{\text{si}}| \qquad (48)$$

$$\mathcal{U}_{\text{so}} = |\mathcal{E}_{\text{so}}| \qquad (49)$$

$$\mathcal{U} = |\mathcal{E}_{\text{ei}}| + |\mathcal{E}_{\text{eo}}| + |\mathcal{E}_{\text{si}}| + |\mathcal{E}_{\text{so}}| \qquad (50)$$

This successful approach of accounting for all of the energies resolves a current issue in physics—how “potential energy internal to a particle system come[s] into the picture” (reference [1] of Part 1)—by treating the electron as a system of potential energies and real rest-mass energies.

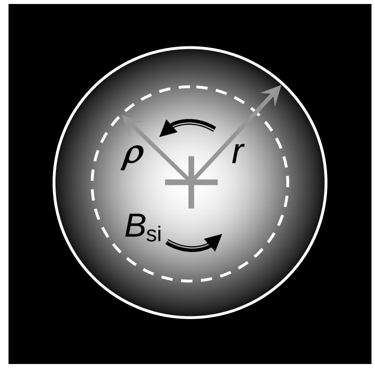
Outside Potential Energy. Part 1 equation (31) gives the *outside* magnetic potential energy $\mathcal{E}_{\text{so}}$, and Part 1 equation (35) gives the *outside* electric potential energy $\mathcal{E}_{\text{eo}}$. But

[2] Subscript notation for the energy variables is explained in Part 1.

the equations for electric potential energy $\mathcal{E}_{\text{ei}}$ *inside* the electron and magnetic potential energy $\mathcal{E}_{\text{si}}$ *inside* the electron are both functions of the charge density and depend upon the *b*-coefficients that enter into the Laurent series established in Part 1. The *b*-coefficients will be evaluated below. See endnote [24] for an explanation of the subscript notation of the *b*-coefficients.

Figure 6. Cross Section of the Ring.

An inside magnetic $\mathbf{B}_{\text{si}}$ field is obtained from Ampere's Circuital Law applied to the geometry of Figure 6. Spin Current $I_{s\rho}$ is a function of ρ and has the direction of a vector coming out of the paper and toward the reader. Direction of the vector $\mathbf{B}_{\text{si}}$ field is shown by the two arrows.



Inside Magnetic Field B_{si}. Setting magnetic permeability $\mu = \mu_{\text{o}}$, where μ_{o} is the magnetic permeability of "free space", setting charge velocity $v_{\text{s}} = c$, and substitution of the charge distribution of equation (20) as a function of radius ρ into equation (17) yields the magnetic field inside the ring (see Figures 5a and 5b in Part 1):

$$B_{\text{si}} = \frac{\left(c\mu_{\text{o}}\left(q(6r-3\rho)+\pi^2 R(r-\rho)^2\left(4b_1 r+3b_2(2r+\rho)\right)\right)\right)}{(12\pi^2 r^2 R)} \qquad (51)$$

Inside Magnetic Potential Energy Density $\mathcal{E}_{\textbf{si density}}$. Energy density is found by using equation (51):

$$\mathcal{E}_{\text{si density}} = \frac{B_{\text{si}}^2}{2\mu_{\text{o}}}$$

$$= -\frac{\left(c^2\mu_{\text{o}}\left(-3q(-2+\rho_r)+\pi^2 r^2 R(-1+\rho_r)^2\left(4b_1+3b_2(2+\rho_r)\right)\right)^2\right)}{(288\pi^4 r^2 R^2)} \qquad (52)$$

where ρ has been replaced by $r \cdot \rho_r$ in order to scale and display the smaller radius ρ_r to range from zero to one.

Magnetic Potential energy inside the ring $\mathcal{E}_{\textbf{si}}$ is obtained by integrating the density over the volume of the ring:

$$\mathcal{E}_{\text{si}} = \int_{\rho_r=0}^{1} \mathcal{E}_{\text{si density}}\,(r^2)(2\pi R)\,(2\pi\rho_r)d\rho_r$$

$$= -\frac{c^2\mu_{\text{o}}(6930q^2+84(32b_1+57b_2)\pi^2 q r^2 R+(448b_1^2+1536b_1 b_2+1323b_2^2)\pi^4 r^4 R^2)}{(60480\pi^2 R)} \qquad (53)$$

where the sign is chosen for *negative* magnetic potential energy according to the Rule of Signs (see Part 1) and the *b*-coefficients are yet to be determined.

From equation (48), the inside magnetic self-energy is

$$\mathcal{U}_{\mathrm{si}} = \frac{c^2\mu_{\mathrm{o}}(6930q^2 + 84(32b_1 + 57b_2)\pi^2 q r^2 R + (448b_1^2 + 1536b_1 b_2 + 1323b_2^2)\pi^4 r^4 R^2)}{(60480\pi^2 R)} \qquad (54)$$

Combined Magnetic Potential Energy of the Ring $\boldsymbol{\mathcal{E}_{\mathrm{s}}}$ is obtained by adding the inside and outside magnetic potential energies:

$$\mathcal{E}_{\mathrm{s}} = \mathcal{E}_{\mathrm{si}} + \mathcal{E}_{\mathrm{so}} \qquad (55)$$

$$= -\frac{\left(c^2\mu_{\mathrm{o}}\left(-8190q^2 + 84(32b_1 + 57b_2)\pi^2 q r^2 R + (448b_1^2 + 1536b_1 b_2 + 1323b_2^2)\pi^4 r^4 R^2 + 7560q^2 \mathrm{Log}\left[\frac{8R}{r}\right]\right)\right)}{(60480\pi^2 R\ )}$$

Combined Magnetic Self-Energy of the Ring $\boldsymbol{\mathcal{U}_{\mathrm{s}}}$ is obtained by adding the inside and outside magnetic self-energies:

$$\mathcal{U}_{\mathrm{s}} = \mathcal{U}_{\mathrm{si}} + \mathcal{U}_{\mathrm{so}} = -\mathcal{E}_{\mathrm{s}} \qquad (56)$$

$$= \frac{\left(c^2\mu_{\mathrm{o}}\left(-8190q^2 + 84(32b_1 + 57b_2)\pi^2 q r^2 R + (448b_1^2 + 1536b_1 b_2 + 1323b_2^2)\pi^4 r^4 R^2 + 7560q^2 \mathrm{Log}\left[\frac{8R}{r}\right]\right)\right)}{(60480\pi^2 R\ )}$$

Angular momentum of an electromagnetic object. Equation (56) gives the 'total magnetic field energy of the model.' The angular momentum p_{s} is then defined by the 'rotational kinetic energy' or the 'inductive energy' of the magnetic field [25, p. 278]. In particular, the angular momentum (spin) p_{s} is the result of the ring's magnetic field energy:

$$p_{\mathrm{s}} \equiv \mathcal{M}_S R c \qquad (57)$$

where $\mathcal{M}_s = \mathcal{U}_{\mathrm{s}}/c^2$ and c is the velocity of charge moving along the ring's circumference.

Solving for the *b*-coefficients. Combining equations (56) and (57) and setting electron spin[3] to $p_{\mathrm{s}} = \hbar/2$ yields a quadratic equation with two roots of b_1 which are designated b_{11} and b_{12}.

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[3] There is a theoretical and empirical basis for setting the electron spin to one-half the value of Planck's constant divided by 2π [16] (which is related to the fact that the electrical and magnetic (spin-related) forces are each ½ of the total force holding the particle together).

$$b_{11} = -\frac{1}{56\, c\mu_o \pi^4 r^4 R^2} \times 3\left(8\, c\mu_o \pi^2 r^2 R\, (7\, q + 4\, b_2 \pi^2 r^2 R) + \sqrt{\left(-c\mu_o \pi^4 r^4 R^2 \left(-23520\, \hbar\pi^2 + c\mu_o(-9506\, q^2 + 140\, b_2 \pi^2 q r^2 R + 5\, b_2^2 \pi^4 r^4 R^2) + 5880\, c\mu_o q^2 \mathrm{Log}\left[\frac{8R}{r}\right]\right)\right)}\right) \quad (58)$$

$$b_{12} = \frac{1}{56\, c\mu_o \pi^4 r^4 R^2} \times 3\left(-8\, c\mu_o \pi^2 r^2 R\, (7\, q + 4\, b_2 \pi^2 r^2 R) + \sqrt{\left(-c\mu_o \pi^4 r^4 R^2 \left(-23520\, \hbar\pi^2 + c\mu_o(-9506\, q^2 + 140\, b_2 \pi^2 q r^2 R + 5\, b_2^2 \pi^4 r^4 R^2) + 5880\, c\mu_o q^2 \mathrm{Log}\left[\frac{8R}{r}\right]\right)\right)}\right) \quad (59)$$

Equations (58) and (59) are two possible solutions of the b_1 coefficient. We selected the negative solution of (58) because it leads to the negative charge of the *electron* charge density $\lambda[\rho]$.

$$b_1 = b_{11} \quad (60)$$

Electric Potential Energy Using b_{11}. The electric field intensity E inside the ring was derived and displayed as equation (22) in Part 1. Using this intensity, the density of inside electric potential energy is obtained:

$$\mathcal{E}_{\text{ei density}} = \frac{\epsilon_o E^2}{2} = \frac{\mu_o c^2 q_\rho^2}{32\pi^2 R^2 \rho^2} \quad (61)$$

where ϵ_o has been replaced by $1/\mu_o c^2$ to facilitate combination of electric and magnetic equations.

Inserting equations (58) and (60), replacing ρ with $r \cdot \rho_r$ and replacing q_ρ with $q_{\rho\, DV}$ in equation (44) gives the density of the inside electric potential energy density. Integrating this density over the volume of the ring yields the inside electric potential energy:

$$\mathcal{E}_{\text{ei}} = \int_0^1 \left(\mathcal{E}_{\text{ei density}}\right)^2 r^2\, (2\pi R)\, (2\pi\rho_r)\, d\rho_r$$
$$= -\frac{c(-4\hbar\pi^2 - 2c\mu_o q^2) + c\mu_o q^2 \mathrm{Log}\left[\frac{8R}{r}\right]}{8\pi^2 R} \quad (62)$$

Magnetic potential energy $\mathcal{E}_{\mathrm{si}}$ inside the ring. From equations (53) and (58):

$$\mathcal{E}_{\mathrm{si}} = \frac{c(-4\hbar\pi^2 - 2c\mu_{\mathrm{o}}q^2) + c\mu_{\mathrm{o}}q^2 \mathrm{Log}\left[\frac{8R}{r}\right]}{8\pi^2 R} = -\,\mathcal{E}_{\mathrm{ei}} \qquad (63)$$

Using electron mass $\mathcal{M}_{\mathrm{e}}$ to solve for Radius *R*. The empirical value of electron mass used is well accepted and given as a fundamental physical constant by CODATA 2006 [Part 1, ref. 16]. Evans assumes that 'rest-mass' is an invariant property of the electron:

> "We assume throughout that if a neutron, proton, electron, neutrino or meson enters a nucleus, the particle retains its identity and extra nuclear characteristics of spin, statistics, magnetic moment and rest mass." [26, p. 277].

But the SCR model reveals that *electron mass is not, and cannot be treated as, a constant under all conditions.* To do so is a fundamental error that has needlessly contributed to the invention of "solutions" (e.g., the declaration of new 'particles') that do not in reality exist—except as unstable exploding charge fragments with short existence. Rather, electron mass *changes* whenever an electron:

- is accelerated,
- binds to one or more other charged particles, or
- is (somehow) energized to an excited energy state.

First, consider the case of a free electron that remains in the *ground state* when some ionized molecule comes close enough to bind the electron to the molecule. As will be shown at equation (66), the SCR model further reveals that *the electron's radius R is not 'forever fixed' but depends upon the electron's mass-energy $\mathcal{U}$.* In contrast, the Copenhagen interpretation asserts that the properties of an electron do not change, but remain fixed, when a (ground-state) electron enters into a molecule, even though it can remain bound to other particles that constitute the molecule. Using a physical SCR model for the electron—rather than pretending that physical reality consists of mathematical abstractions called 'infinitesimally small point-particles'—*we discover that the electron takes on a previously undiscovered physical form that is durable, deformable, robust and complex*— just as common sense would specify physical reality should be.

With a physical model, therefore, we discover that mass (as we measure it) is not equivalent to "stuff" or physical material. Rather mass is the reaction force we measure and experience when we attempt to accelerate the underlying real physical material that is there: *charge.* Not only is mass not equivalent to material, we discover that mass is not constant under all conditions. Since mass is essentially a force, the force can change depending on a variety of environmental contributions. Similarly, this same physical model (the SCR model) leads us to conclude that the form of the charge (referred to as a 'particle') can also change properties (e.g., radius) in various degrees of freedom giving rise to size changes, various energy states, and radiation/absorption mechanisms.

If electron mass (and its equivalent mass-energy) always remained constant, then the atomic weights of the elements could be predicted from the sum of the weights of constituent particles. But, even for a neutron (composed of one electron and one proton), the mass-energy of a neutron is considerably greater than the combined mass-energy of one proton and one electron. Atomists have traditionally accounted for this extra neutron mass-energy by assuming various additional particles must be present in the neutron (which would be reasonable only if mass and material are the same thing—an erroneous assumption that has persisted for over one hundred years).

But the SCR model, based on classical electrodynamics coupled with a defined physical structure for the particle, accounts for the extra neutron mass-energy as a result of the *mutual* energy that links one proton and one electron together in a "particle" known as the neutron. We will show in equation (66) the SCR model's prediction of the relationship between the mass-energy of an electron and its size, thus providing a physical mechanism for exchanging and storing electromagnetic energy among multiple charged particles. Unlike the current standard model where the electron is assumed to be (and declared fervently by some to in reality be) a "point-particle" having no physical extent, structure, size or shape, the SCR model sensibly conforms to the 'law of conservation of energy' and shows how this law is never violated.

Second, for the case of electrons in *excited energy states,*[4] radius *R* (or its equivalent value of loop length in a helical structure) provides a variable physical mechanism for radiation of energy. As *R* varies, radiation frequencies (or spectral lines) are produced and observed in discrete wavelengths, e.g., such as predicted by the Rydberg equation [28].

The total rest-mass energy $\mathcal{U}$ of all electric and magnetic fields is the energy equivalent of electron mass multiplied by the square of the speed of light as in equations (45 and 50):

$$\begin{aligned}\mathcal{U} &= \mathcal{U}_{\mathrm{ei}} + \mathcal{U}_{\mathrm{eo}} + \mathcal{U}_{\mathrm{si}} + \mathcal{U}_{\mathrm{so}} \\ &= \frac{1}{60480\pi^2 R} c(30240\hbar\pi^2 \\ &\qquad + c\mu_{\mathrm{o}}(6930q^2 + 84(32b_1 + 57b_2)\pi^2 q r^2 R \\ &\qquad + (448b_1^2 + 1536b_1 b_2 + 1323b_2^2)\pi^4 r^4 R^2) \\ &\qquad + 7560c\mu_{\mathrm{o}} q^2 \mathrm{Log}[8R/r]) \qquad (64)\end{aligned}$$

In equation (64) replace b_1 with b_{11} (the first root of $b_1$as given by equation (58)) to obtain the energy formula $\mathcal{U}_{b11}$ of the electron:

[4] to be considered in another paper on electrons in excited energy states

$$\mathcal{U}_{b11} = \frac{c(4\hbar\pi^2 + c\mu_o q^2)}{4\pi^2 R} \qquad (65)$$

With $\mathcal{U}_{\text{b11}}$ set to the electron rest-mass energy $\mathcal{U}$, solve equation (65) for the electron radius *R:*

$$R = \frac{c(4\hbar\pi^2 + c\mu_o q^2)}{4\pi^2 \mathcal{U}} = 3.8705624254267599063 \times 10^{-13} \text{ meters} \qquad (66)$$

Equation (66) is the equation of the radius *R* of a free and unexcited electron in the ground energy state. The empirical value of the electron rest-mass energy $\mathcal{U}$ and some physical constants were used to compute the dimension of an electron: $R = 3.8705624254267599063 \times 10^{-13}$ meters. This is the actual value of the large radius *R* for an unexcited and free electron. *Thus, the SCR Model predicts the radius R of a free electron is finite and physically related to electron properties such as its rest-mass, spin, magnetic moment, wavelength, and line spectra.*

Magnetic moment. The spinning charged ring model predicts the magnetic moment μ_{e} of a free electron. The magnetic moment of a loop of current is defined as the electrical current times the area enclosed by the loop. For one unit of charge moving in a circular loop, the magnetic moment of the loop is

$$\mu_{\text{e}} = \frac{c\ e\ R}{2} = -9.2955509905057462369 \times 10^{-24} \text{ Joule/Tesla} \qquad (67)$$

where *c* is the velocity of the moving charge *e*. According to the SCR model, this is the actual value of the magnetic moment for an unexcited and free electron.

According to Quantum Mechanics (QM), the electron magnetic moment is *anomalous* and equal to $-9.28476377(23) \times 10^{-24}$ Joule/Tesla [16]. The magnetic moment is called “anomalous” because it deviates from the value obtained by the use of equation (67). Equation (67) cannot be used by Quantum Theory (QT) since the quantum electron is said to be a point-like particle with a radius *R* = 0. Instead, QT has produced a complicated explanation for the anomaly that is based in some QM assumptions about elementary particles (including the electron).

Thickness of the ring. An equation for the small radius *r* of the ground-state electron can be found in a 1990 paper by Bergman and Wesley [27]:

$$r = \frac{8\hbar}{\mathcal{M}c} \exp\left(\frac{-\pi}{\alpha} - \frac{1}{2}\right) \qquad (68)$$

where α is the fine-structure constant. Equation (68) was derived from Lenz’s Law, an equation for the Compton wavelength, and the law of conservation of energy [27].

FINDING THE CHARGE DISTRIBUTION

To find a charge distribution that is consistent with the balance of forces holding the electron together, we must solve for the *b*-coefficients:

$\boldsymbol{b_{11}}$. Select the first solution of b_1 (which pertains to the electron). Then, $b_1 = b_{11}$ as given by equation (58).

$\boldsymbol{b_{01}}$. Starting with Part 1 equation (42), enter the value of radius *R* found at equation (66) and the values of the fundamental physical constants in order to obtain b_{01}:

$$b_{01} = -\frac{4}{3}\, b_{11} - \frac{3}{2}\, b_{21} + \frac{2.097039141403234 \times 10^{-8}}{r^2} \qquad (69)$$

Residue $\boldsymbol{b_{\mathrm{res1}}}$. From Part 1 equation (37),

$$b_{\mathrm{res1}} = -b_{01} - b_{11} - b_{21} \qquad (37)$$

Charge density $\boldsymbol{\lambda[\rho]}$. From Part 1 equation (18), the distribution of charge in the ring interior is

$$\lambda[\rho] = \frac{r\, b_{\mathrm{res}}}{\rho} + b_0 + \frac{b_1\, \rho}{r} + \frac{b_2\, \rho^2}{r^2} \qquad (18)$$

Then the derivative of $\lambda[\rho]$ taken with respect to ρ is

$$\lambda' = \frac{b_1}{r} - \frac{b_{\mathrm{res}}}{\rho^2} + \frac{2 b_2 \rho}{r^2} \qquad (70)$$

Substituting equation (37) for b_{res} yields

$$\lambda'_{\rho=r} = \frac{b_0 + 2b_1 + 3b_2}{r} \qquad (71)$$

The first solution of b_0 is selected, i.e., b_{01}. Thus set $b_0 = b_{01}$ from equation (69) into equation (71). And, select $b_1 = b_{11}$ as in equation (60) which pertains to electrons. These conditions give:

$$\lambda'_{\rho=r} =$$

$$\frac{2.09704 \times 10^{-8} - \frac{0.202642\, q}{R}}{r^3} + \frac{0.357143\, b_2}{r} - \frac{1}{c\, r^5\, R^2\, \mu_o}\, 0.00361861 \sqrt{-c\, r^4\, R^2\, \mu_o \left(-232133.\, \hbar + 5880.\, c\, q^2\, \mathrm{Log}\left[\frac{8.\, R}{r}\right] \mu_o + c\left(-9506.\, q^2 + 1381.74\, q\, r^2\, R\, b_2 + 487.045\, r^4\, R^2\, b_2^2\right) \mu_o\right)} \qquad (72)$$

EVALUATION OF THE *b*-COEFFICIENTS

Normalized and Dimensionless Variables. In some of the following variable labels, *N* indicates that large values are normalized to a positive unit value (1), e.g., *NCD* is the normalized, average, volume charge density *CD*. And *DV* indicates that the independant variables are also dimensionless variables formulated according to the method of Professor Otto Ruehr.

Charge Density inside the Ring. Average charge density *CD* is the charge divided by the ring volume:

$$CD = \frac{-q}{\left((2\pi R)(\pi r^2)\right)} \qquad (73)$$

And the normalized charge density *NCD* is:

$$NCD = \frac{CD}{CD} = 1 \quad (74)$$

Since the electron charge q is negative, we make *CD* positive by inserting the negative sign into equation (73) in order to retain the signs computed for the normalized *b*-coefficients shown below.

Solving for the b_2-coefficients. At the surface of the ring, the change of charge density as a function of radius ρ becomes zero, so at the boundary (i.e. at $\rho = r$), $\lambda[\rho]$ is continuous since there is assumed to be no charge outside the ring model. Therefore, to help insure smoothness in $\lambda[\rho]$ at the point $\rho = r$, we set $\lambda'_{\rho=r} = 0$, and then we solve for b_{21} by obtaining the two solutions of the quadratic equation resulting from this setting $\lambda'_{\rho=r} = 0$, and then solving the result for b_{211}:

$\boldsymbol{b_{211}} =$

$$-\frac{1}{c\, r^4 R^2 \mu_o} 1.03666\times 10^{-104} \left(690.872\, c\, r^2 R \left(-6.60195\times 10^{100}\, q + 7.80801\times 10^{93}\, R\right) \mu_o + 82.575 \sqrt{\left(-c\, r^4 R^2 \mu_o \left(-3.09725\times 10^{205}\, \hbar - 53.\, c \left(2.17929\times 10^{202}\, q^2 + 2.72329\times 10^{194}\, q\, R - 4.02599\times 10^{186}\, R^2\right) \mu_o + 7.84543\times 10^{203}\, c\, q^2\, \mathrm{Log}\left[\frac{8.\, R}{r}\right] \mu_o\right)\right)}\right) \qquad (75)$$

Insert the fundamental constants and ring parameters and divide by the charge density *CD* to get the normalized numerical value of Nb_{211}:

$$Nb_{211} = -26.385650733956960 \qquad (75a)$$

$\boldsymbol{b_{212}} =$

$$\frac{1}{c\, r^4 R^2 \mu_o} 1.03666\times10^{-104} \left(-690.872\, c\, r^2 R \left(-6.60195\times10^{100} q + 7.80801\times10^{93} R\right)\mu_o + 82.575\sqrt{\left(-c\, r^4 R^2 \mu_o \left(-3.09725\times10^{205} \hbar - 53.\, c\left(2.17929\times10^{202} q^2 + 2.72329\times10^{194} q R - 4.02599\times10^{186} R^2\right)\mu_o + 7.84543\times10^{203} c\, q^2 \operatorname{Log}\left[\frac{8.\,R}{r}\right]\mu_o\right)\right)}\right) \quad (76)$$

Insert the fundamental constants and ring parameters and divide by the charge density *CD* to get the normalized numerical value of Nb_{212}:

$$Nb_{212} = 2.385650733956960 \quad (76a)$$

Solving for the remaining *b*-coefficients.

$\boldsymbol{b_{111}}$. In equation (58), replace b_2 with b_{211} to get b_{111}. Insert the fundamental constants and ring parameters and divide by the charge density *CD* to get the normalized numerical value of Nb_{111}:

$$Nb_{111} = 44.59737407930642 \quad (77)$$

$\boldsymbol{b_{112}}$. In equation (58), replace b_2 with b_{212} to get b_{112}. Insert the fundamental constants and ring parameters to get the normalized numerical value Nb_{112}:

$$Nb_{112} = -6.867714151403160 \quad (78)$$

$\boldsymbol{b_{011}}$. In equation (69), replace b_1 with b_{111} and b_2 with b_{211} to get $b_{011.}$ Insert the fundamental constants and ring parameters to get the normalized numerical value Nb_{011}:

$$Nb_{011} = -18.88468933813978 \quad (79)$$

$\boldsymbol{b_{012}}$. In equation (69), replace b_1 with b_{112} and b_2 with b_{212} to get b_{012}. Insert the fundamental constants and ring parameters to get the normalized numerical value Nb_{012}:

$$Nb_{012} = 6.578476100935440 \quad (80)$$

$\boldsymbol{b_{\mathrm{res11}}}$. In equation (37), replace b_0 with b_{011} and b_1 with b_{111} and b_2 with b_{211} to get b_{res11}. Insert the fundamental constants and ring parameters to get the normalized numerical value Nb_{res1}:

$$Nb_{\mathrm{res11}} = 0.6729659927903257 \quad (81)$$

$\boldsymbol{b}_{\mathbf{res12}}$. In equation (37), replace b_0 with b_{012} and b_1 with b_{112} and b_2 with b_{212} to get b_{res12}. Insert the fundamental constants and ring parameters to get the normalized numerical value Nb_{res12}:

$$Nb_{\mathrm{res12}} = -2.0964126834892400 \qquad (82)$$

Equations (81) and (82) are two possible solutions to the Nb_{res} coefficient. We selected the negative solution of (82) because it leads to the negative charge of the *electron* charge density $\lambda[\rho]$.

Volume charge density. Dividing *charge density* as given by equation (18) found in Part 1 by the *average charge density* as given by equation (73), and selecting the *b*-coefficients associated with electrons, namely $b_{\mathrm{res}} = b_{\mathrm{res12}}$, $b_0 = b_{012}$, $b_1 = b_{112}$, $b_2 = b_{212}$, and then inserting the numerical values found above for the physical dimensions of the electron, and then inserting the fundamental physical constants, and finally replacing the variable ρ with $r \cdot \rho_r$, we compute the normalized dimensionless equation of the charge density NDV$\lambda_{\rho 12}$ as a function of radius ρ:

$$\mathrm{NDV}\lambda_{\rho 12} = -6.578476100935400 + \frac{2.096412683489240}{\rho_r}$$

$$+ 6.86771415140316\, \rho_{\mathrm{r}} - 2.385650733956960\, \rho_r^2 \qquad (83)$$

Plot of normalized volume charge density, equation (83):

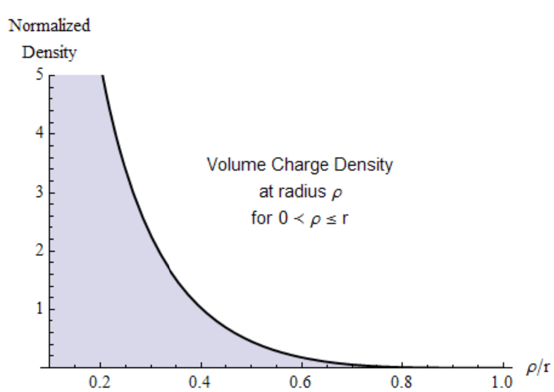

Figure 7.

Normalized, Dimensionless Charge Density plotted as a function of ρ_r.

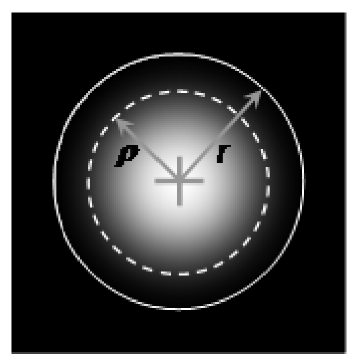

Figure 8. Cross Section of Ring.

Cross-section of ring showing high charge density at the center decreasing to zero at the surface $\rho = r$.

Figure 7 shows a mathematical singularity as ρ approaches zero from above. However, as to the physics here, the line charge density of the circle defined by $\rho = 0$ itself must be zero as may be seen from the following graph, Figure 9, of normalized q_ρ which approaches zero as ρ approaches zero from above. Thus the mathematical singularity is *not* a physical singularity.

Figure 7 applies to the negatively charged electron since we selected $b_{\text{res}12}$ above. We ignored $b_{\text{res}11}$ because it is positive, whence the electron would have to have a positive charge too because b_{res} governs $q_\rho[\rho]$ for ρ very small in the limit.

Distribution of charge inside the ring is given by equation (20) found in Part 1 by selecting the b-coefficients associated with electrons, namely $b_{\text{res}} = b_{\text{res}12}$, $b_0 = b_{012}$, $b_1 = b_{112}$, $b_2 = b_{212}$, and then inserting the numerical values found above for the physical dimensions of the electron, and then inserting the fundamental physical constants, and finally replacing the variable ρ with $r \cdot \rho_r$. These computations yield the normalized dimensionless equation of the charge $NDVq_\rho[\rho_r]$ taking values from $0 \leq \rho_r \leq 1$ as a function of the normalized radius ρ_r.

$$NDVq_\rho[\rho_r] = \rho_r\,(4.1928253669784800424 - 6.578476100935440127\,\rho_r + 4.5784761009354401273\,\rho_r^2 - 1.1928253669784800424\,\rho_r^3) \quad (84)$$

Plot of normalized dimensionless charge, equation (84):

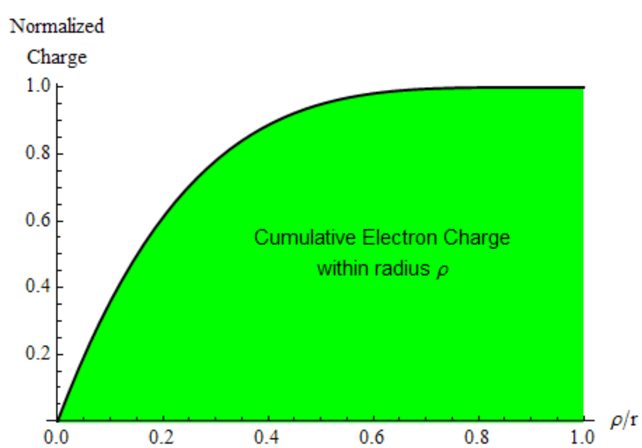

Figure 9
Normalized, Dimensionless,
Cumulative Charge Plotted as a Function of ρ_r.

Summary of Part 2 and Continuation. The distribution of charge inside an electron modeled as a Spinning Charged Ring (SCR) has been derived. The model builds upon the equilibrium of electric and magnetic pressures at the surface of the ring and everywhere inside the ring derived in Part 1, and further illustrates how the electron's mass and size (radius *R*) change under certain conditions. A follow-on paper will show that the model predicts the empirical properties of electrons.

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References and Notes for Part 2. For references [1] through [23] see Part 1.

[24] **Subscript notation for the *b*-coefficients** is as follows**:** The typical *b*-coefficient is b_{ijk}, where i assumes values from the set [0, 1, and 2], j takes on values from [1, 2], and k takes on values from [1, 2] also. The i subscript designates the original Laurent series *b*-coefficient b_{i}, so that b_0, b_1, and b_2 are the three coefficients of the original Laurent series for $\lambda[\rho]$ (we will consider b_{res} below). The subscript j comes from the fact that there are two solutions of the quadratic equation for b_1 and so they naturally have to be denoted by b_{11} and b_{12} to avoid confusion. That then means there are now two separate values for b_0 after substitution of b_{11} and b_{12} in the formula for b_0, where b_{11} and then b_{12} are substituted for b_1 in this formula for it. We denote the results of these two substitutions as b_{01} and b_{02}, respectively (working backwards). Additionally there are two solutions for b_2 for each of b_{11} (and b_{01}) and b_{12} (and b_{02}), that are denoted by b_{211} and b_{212} corresponding to b_{01} and b_{11}, and b_{221} and b_{222} corresponding to b_{02} and b_{12}, respectively.

Now, as to b_{res}, b_{resjk} corresponds to the j and k of the corresponding to b_{ijk}, where the i subscript is missing in b_{resjk} because i refers to the original b_0, b_1, and b_2. And b_{res} itself determines the original Laurent series coefficient (namely the first one) so the subscript i is not needed. Thus b_{res11} corresponds to b_{011}, b_{111} and b_{211}; b_{res12} corresponds to b_{012}, b_{112} and b_{212}; b_{res21} corresponds to b_{021}, b_{112}, and b_{221}; and b_{res22} corresponds to b_{022}, b_{122} and b_{222}. However, since the algebraic sign of b_{resjk} determines the sign of $\lambda[\rho]$ very near zero, the sign of it must be negative, and this eliminates b_{res11} but not b_{res12}.

[25] J. P. Wesley, **Advanced Fundamental Physics**, pub. by Benjamin Wesley, 1991.

[26] R. D. Evans, **The Atomic Nucleus**, Krieger Publishing Company, 1955.

[27] D. L. Bergman and J. P. Wesley, "Spinning Charged Ring Model of Electron," *Galilean Electrodynamics* **1** (5), equation (49) 1990. Also, see the explanation for the *r*-equation given by Wesley in reference [25].

[28] J. Lucas and C. W. Lucas, Jr., "A Physical Model for Atoms and Nuclei—Part 3," *Foundations of Science* **6** (1), published by Common Sense Science, February, 2009.

(To be continued in Part 3.)