The Electrodynamic Origin of the Force of Inertia ($F = m_i a$)—Part 1

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Abstract. A review of Newton's *Principia* shows the significance of his Existence Theorem for absolute space and time in order to explain the force of inertia and the centrifugal force. A review of the history of Einstein's General Theory of Relativity reveals the failure to establish the basis of inertia and the centrifugal force in terms of relative coordinates as Mach had envisioned. In this work the force of inertia including the centrifugal force is based on relative coordinates. It is shown to be an average residual force due to the acceleration terms in the derived universal electrodynamic force between vibrating neutral dipoles consisting of atomic electrons vibrating with respect to protons in the nucleus of neutral atoms. The inertial mass is derived and shown to be equal to the gravitational mass. The vibrational mechanism for both gravitational and inertial mass causes the magnitude of both masses to decay over time. The inertial force has a non-radial $\mathbf{R} \times (\mathbf{R} \times \mathbf{A})$ term which makes possible certain observed non-Newtonian inertial gyroscopic motions. Arguments are made that this derived law of inertia is superior to Newton's Law of Inertia $(\mathbf{F} = m\mathbf{a})$, because it is properly based on relative coordinates, contains a second term that describes additional observed phenomena, and contains relativistic type corrections for high velocity. Also it is superior to Einstein's field equations of General Relativity Theory which have never been able to explain the force of inertia or the centrifugal force in terms of relative coordinates nor the phenomena predicted by the second term of this work.

Review of Newton's Principia. Newton (1687) was the first to explain the forces of nature in great detail in his book *Mathematical Principles of Natural Philosophy* or *Principia* for short [1]. In that famous work Newton developed and presented the laws of mechanics, the force of inertia, the centrifugal force, and the force of gravity. Regarding the origin of gravity Newton said in the *General Scholium* at the end of his book:

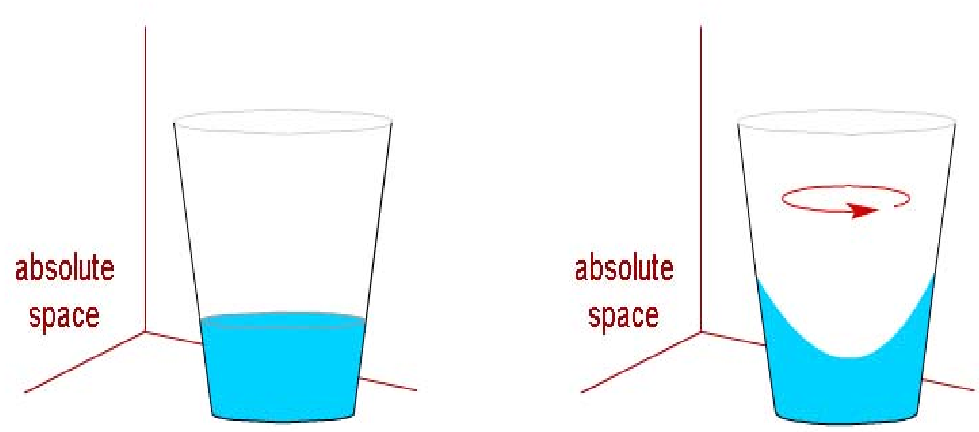
> But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy [1, *p.* 371].

Newton is often criticized for inventing ad-hoc the concepts of absolute space and time. However, Newton did not invent these concepts, but merely adopted the universal familiar notions validated by millennia of experience, thought and reasoning. In his *Principia* Newton states at the beginning of the book in the Scholium following the Definitions [1, *p.* 8],

> I do not define time, space, place, and motion as being well known to all.

Newton merely explained and defined these concepts formally and rigorously.

What is little appreciated is that Newton did not merely adopt the concepts of absolute time and absolute space as convenient assumptions or hypotheses as many scientists do today. Rather, in keeping with his notions of experimental philosophy, he proved an “Existence Theorem” for absolute space. This was done with his rotating water pail experiment. Newton believed that this experiment proved the existence of absolute accelerations. From logic one can show that the existence of absolute accelerations proves the existence of absolute space. The significance of Newton’s experiment and the logical arguments constituting his “Existence Theorem” are not generally appreciated. Thus Newton’s adoption of absolute space was not a mere hypothesis, assumption or premise, but rather claimed as an experimentally demonstrable feature of nature.



The details of Newton’s water pail experiment are summarized in the table below.

Stage	***Pail Rotation***	***Water Rotation***	***Relative Rotation***	***Water Concavity***
1	No	No	No	No
2	Yes	No	Yes	No
3	Yes	Yes	No	Yes
4	No	Yes	Yes	Yes

The proof of the “Existence Theorem” for absolute space goes as follows where accelerations are responsible for the concave surface of the water in the pail:

1. In the rotating water pail experiment, the concavity of the water surface can occur whether there is relative motion (Stage 4) or no relative motion (Stage 3).

2. The concavity occurs whenever the water rotates (Stages 3 and 4) and the concavity does not occur whenever the water does not rotate (Stage 1 and 2) irrespective of relative rotation.

Therefore concavity is caused by the true or real absolute rotation of the water. Therefore real or absolute accelerations exist.

Using logic one can show that if acceleration is always absolute experimentally, then all of space must be absolute including velocities. One can also show from the Galilean transformation that if space is absolute time must also be absolute. In this manner Newton's rotating pail experiment was claimed to show the existence of absolute space and time.

Note that this proof is flawed, because it assumes that the effect of the small thin pail is more significant than that of the earth, moon, and sun. If Newton had looked at the water surface at the time of the rising of the moon or the sun, he could have observed a small tide in the water. Of course, the effects of the earth would be much greater, because it is bigger and much closer. Thus what Newton really showed was that there must be relative acceleration with respect to the most significant massive body, the earth. Since the earth spins on its axis and orbits the sun which orbits the center of the Milky Way, these accelerations with respect to absolute space should have been observed and they were not.

Review of Einstein's General Relativity Theory. The first significant challenge to Newton's Existence Theorem for absolute space was attempted 200 years later by Ernst Mach, the founding father of modern Positivist ideology which was strongly opposed to the notion of absolute space and the aether. The key to refuting Newton's Existence theorem is related to the source of inertia. In Newton's physics, the source of inertia is absolute space. For Newton the inertia is the resistance to a change of state against absolute space. If the notion of absolute space is to be discredited, then there must be some other source of inertia. Mach asserted that inertia arises solely from the action of the other bodies and is purely relational. Mach (1893) tried to formulate a theory for this purpose [2] but, as noted by Einstein (1921) [3, *p*. 248], it was a complete failure.

1. Problem of Relational Inertia for Relativity Theory. What is necessary to refute Newton's Existence Theorem is a purely relational theory of inertia providing a satisfactory account of mechanics. Einstein (1915) attempted to do just that with his General Relativity Theory [3]. The success or failure of General Relativity Theory in this regard would be determined by whether it provided a purely relational theory of inertia.

Einstein's avowed objective in creating General Relativity Theory was to obtain a relational theory of inertial forces not requiring absolute space. His objective was not a superior theory of gravitation. There was no pressing need for a new theory of gravitation. Newton's theory of gravitation was adequate at that time for all known gravitational phenomena. However, Einstein came to realize that inertia and gravitation are so intimately connected that a theory of relative inertia must also be related to the theory of gravitation. Thus it was only incidentally that Einstein's theory of general relativity became concerned with gravitation. The primary concern was inertia.

The basic field equations of General Relativity Theory are

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -\frac{8\pi}{c^4} G\, T_{\mu\nu} \qquad (1)$$

where R and g pertain to the structure of space-time and T represents the distribution of matter. These equations correspond to the well-known Poisson's equations of Newtonian theory.

Einstein (1917) said,

> In a consequential theory of relativity there can be no inertia of matter against space but only inertia of matter against matter [3].

Einstein was also careful to point out that the postulate of relativity of inertia cannot be satisfied by merely admitting that the other bodies influence inertia, but demands that they should *entirely* cause it. The corollary is that if there were no other bodies in the universe there would be no inertia. This is an absolute necessity for a relational theory of inertia, and Einstein was careful to state it explicitly.

Mathematically this criterion for a relational theory of inertia requires that for $T_{\mu\nu} = 0$ everywhere, Einstein's equations should have no solutions. If the equations admit any solutions for the case of an empty universe, then the theory fails as a theory of relational inertia, and therefore fails to overthrow Newton's Existence Theorem. Even if the General Relativity Theory were to have great merits as a theory of gravitation, it would still fail as a relative inertia theory if it allows solutions for an empty universe.

It soon became clear that the Einstein field equations must admit solutions for an empty universe, because constant equations for an empty universe have rates of change that are always zero allowing no solutions for a non-empty universe. To remedy the situation Einstein (1917) added a relative inertia term on the left side of the equations which are known as the lambda terms [3]. One effect of this term is to cause the universe to expand.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = -\frac{8\pi}{c^4} G\, T_{\mu\nu} \qquad (2)$$

With this addition Einstein hoped that his equations would no longer admit those inconvenient solutions with inertia for an empty universe, but he was mistaken.

In 1917 the astronomer De Sitter obtained the general solutions for the modified Einstein equations for an empty universe [4]. There was still inertia present. Einstein could not think of any other way to modify his equations of General Relativity Theory and eventually abandoned the idea of contesting Newton's Existence Theorem for absolute space. Thus the notion of absolute space still stands. This failure of Einstein to accomplish his goal in creating General Relativity Theory is no longer emphasized in the scientific literature.

Problem of Rotation for Relativity Theory. In addition to the problem of not being able to build a theory of inertia based on relativity, there is the other problem of explaining rotation. As Eddington (1920) says [5, *p.* 152],

> The great stumbling block for a philosophy which denies absolute space is the experimental detection of absolute rotation.

Any relative space theory must be able to account for the dynamical effects of rotation without an appeal to absolute space.

Newton's concept of absolute space, although well understood by the common man, was always distasteful to philosophers. Even during Newton's life, Bishop Berkley had tried to refute Newton by speculating that the origin of the centrifugal forces, giving rise to the concavity of the water surface in Newton's experiment, results from the rotation relative to the distant stars. According to this view, if the distant stars, the Milky Way, or the universe should rotate (which modern astronomers believe they do), then there should also arise a centrifugal force imparting a concavity to the surface of the water without the water itself rotating with respect to the earth. (Note that there is a point that is missed in these arguments. Everything that we measure is very near to the earth. From Newton's empirical laws for gravity the forces fall off as $1/R^2$. Thus the effect of the sun, moon, other planets, and distant stars is very small compared to that of the earth.)

Einstein hoped that the solutions of his equations of General Relativity Theory would give rise to an appropriate centrifugal force field. As for the case of relative inertia, there is the widespread misconception in the scientific community that Einstein accomplished this with his theory of gravitation. The source of this misconception was a remarkable approximate solution obtained by H. Thirring (1918) [6]. But there was a catch in it. C. Moller (1952) gives some of the details [7, *p.* 433].

> On the other hand, the scalar potential has a form that, besides the usual centrifugal force, gives rise to a non-vanishing component of the force along the axis of rotation.

Unfortunately for Einstein, this axial force term is not observed in nature.

The result of Thirring does not justify the overthrow of Newton's Existence Theorem, because of this unobserved axial force term. Einstein hoped that the exact solution of the full set of equations for General Relativity theory could eliminate the undesirable axial force while leaving the radial centrifugal force as required.

Thirty years later, the eminent logician Kurt Gödel (1949) [8] obtained a solution of the full set of Einstein equations, including the lambda term for a rotating universe, but there was now no centrifugal force field! In the 1960's additional solutions of Einstein's equations were obtained for a spinning universe without the lambda term, but no centrifugal forces emerged. Thus Gödel and others showed that Einstein's General Relativity Theory failed to explain both inertia and rotation in terms of relative coordinates.

Since the time of Newton, the science of electrodynamics has arisen. It now provides a second Existence Theorem for absolute space as follows:

> An electrically charged body radiates as a result of absolute acceleration only, and not as a result of apparent, or virtual, accelerations as seen from different reference frames. Therefore absolute acceleration and motion exist. Therefore absolute space exists.

(Again, one needs to note that this data is also taken on the surface of the earth, where the effect of the earth dominates over all other sources.)

Problem of Inertial and Gravitational Mass. The inertial mass of a particle is defined by Newton's second law $F = m_i a$ (Newton, 1687) [1] and by related mechanical quantities such as momentum $p = m_i v$. The mass m_i is termed inertial, since it is a measure of the persistence of the particle in its current state caused by its interactions with the rest of the universe.

The gravitational mass m_g is defined by Newton's Universal Gravitational Force Law $F_g = G m_{g1} m_{g2}/r_{12}^2$ and by related gravitational quantities such as an object's weight $w = m_g g$. The gravitational mass m_g is a measure of that physical property of a particle that gives rise to the gravitational force. In another paper [9] the origin of the gravitational force was found to be due to the $(v/c)^4$ terms in the electrical force between vibrating neutral dipoles composed of electrons and protons in atoms.

A particle's gravitational mass can be defined and measured by standard operational procedures. Suppose particles 1 and 2 with gravitational masses m_{g1} and m_{g2} are brought to the same location on the surface of the earth. Then the gravitational forces on each, F_{g1} and F_{g2}, are measured by weighing the two objects. Now by definition the ratio of the respective gravitational masses equals the ratio of the corresponding gravitational forces.

$$\frac{F_{g1}}{F_{g2}} = \frac{m_{g1}\, g}{m_{g2}\, g} = \frac{G\, m_{g1}\, m_E / R_E^{\,2}}{G\, m_{g2}\, m_E / R_E^{\,2}} = \frac{m_{g1}}{m_{g2}} \tag{3}$$

Suppose now that particles 1 and 2 are dropped at the same location and their accelerations a_1 and a_2 are measured. Both particles are found to have the same acceleration g. Thus the ratio of their inertial forces F_{i1} and F_{i2} is

$$\frac{F_{i1}}{F_{i2}} = \frac{m_{i1}\, a}{m_{i2}\, a} = \frac{m_{i1}\, g}{m_{i2}\, g} = \frac{m_{i1}}{m_{i2}} \tag{4}$$

Now the inertial and gravitational forces are equal for particles 1 and 2 at rest on the surface of the earth so

$$\frac{F_{g1}}{F_{g2}} = \frac{m_{g1}}{m_{g2}} = \frac{F_{i1}}{F_{i2}} = \frac{m_{i1}}{m_{i2}} \tag{5}$$

By experiment the ratio of the gravitational masses of two particles equals the ratio of the inertial masses by approximately 1 part in 10^{12} [10].

In classical physics the equality of the inertial and gravitational masses is regarded as an extraordinary coincidence. However, in the general theory of relativity by Albert Einstein, this equality is taken as a basic assumption in the "principle of equivalence." The principle of equivalence relates to accelerated reference frames.

An observer in an accelerated reference frame can use Newton's second law to describe a particle's motion. To do so, such an accelerated observer must invoke, in addition to the "real" forces acting on the particle, a fictitious inertial force, $F_i = -m_i\, a_i$. This inertial force depends on the particle's inertial mass m_i and on the acceleration a_i of the observer's reference frame relative to an inertial frame. The non-inertial observer writes Newton's second law as

$$\sum F_{Real} + F_i = m_i\, a \qquad \text{with} \quad F_i = -m_i\, a_i \tag{6}$$

where a is the particle's acceleration as measured in the accelerated frame.

Now if a particle's gravitational mass m_g is precisely the same as its inertial mass m_i, then the inertial force can be written as

$$F_i = -m_i\, a_i = -m_g\, a_i \tag{7}$$

Thus the "fictitious" inertial force is proportional to the observed particle's gravitational mass. This implies that

> An inertial force arising in a non-inertial frame is altogether equivalent to, and indistinguishable from, a gravitational force as perceived by an observer at rest in this accelerated frame [10].

This is Einstein's Principle of Equivalence in his General Theory of Relativity.

According to the Principle of Equivalence there is no experimental way of telling the difference between an inertial acceleration and a gravitational force. Actually this is an idealization based on point particles. In the real world of finite size particles, there is a three dimensional gradient of the gravitational force over the volume of the particle, but a different gradient for an inertial acceleration. Gravity is dependant on the distance between the centers of two objects, and inertia arises solely from the action of other bodies in the universe and is purely relational in origin. Thus, people in elevators can tell the difference between the gradient of the force of gravity and the acceleration due to the motion of the elevator!

Summary Part I. Newton's Second Law ($\mathbf{F} = m\mathbf{a}$) for the force of inertia is based on absolute coordinates and absolute acceleration from his water pail experiments. This conclusion is flawed, since the absolute coordinate system was tied to the earth and not the center of the universe. Mach correctly pointed out that the force of inertia arises solely from the action of other bodies in the universe and is purely relational in origin. Einstein tried to correct the deficiencies of his Special Relativity Theory (SRT) when he developed his General Relativity Theory (GRT). However, all versions of Einstein's field equations of GRT, with and without the lambda terms, failed to properly describe the force of inertia and the centrifugal force in terms of relative coordinates. Neither SRT nor GRT could satisfy Mach's requirement of relative coordinates. Thus neither Newton's Second Law nor Einstein's Relativity Theories could qualify as proper science.

Newton was unable to explain why the inertial and gravitational masses were empirically the same. Gravity and electrodynamics were fundamentally different forces. Einstein could not explain this either. So Einstein created his Principle of Equivalence in General Relativity Theory, where he assumes that they are fundamentally different but equal entities without proof. At best this principle only works for point particles, but all particles in nature have finite size and a non-spherical shape. Thus neither Newton's nor Einstein's approach can explain the fundamental nature of mass and why the gravitational and inertial masses are equal.

In part 2 of this paper a more complete and more satisfying solution to the problems above will be presented in terms of the derived universal electrodynamic force.

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